9.13 The pressure and temperature at the beginning of compression of an air-standard Diesel cycle are 95 kPa and 300 K, respectively. At the end of the heat addition, the pressure is 7.2 MPa and the temperature is 2150 K. Determine

(a) the compression ratio.

(b) the cutoff ratio.

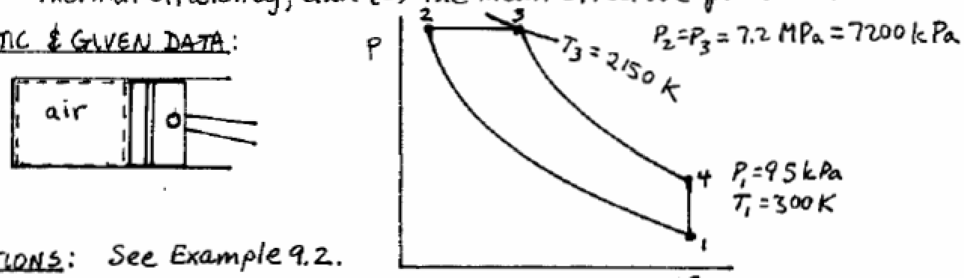
(c) the thermal efficiency of the cycle.

(d) the mean effective pressure, in kPa.

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cut off ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 9.2.

ANALYSIS: Begin by fixing each principal state in the cycle (Table A-22).

State 1: $T_1 = 300$ K, $p_1 = 95$ kPa $\Rightarrow u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$, $p_{r1} = 1.3860$

State 2: For the isentropic compression

$$p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.3860)\left(\frac{7200}{95}\right) = 105.04$$

Thus, $T_2 = 979.6$ K, $v_{r2} = 26.793$, $h_2 = 1022.82$ kJ/kg

State 3: $T_3 = 2150$ K, $p_3 = 7200$ kPa $\Rightarrow h_3 = 2440.3$ kJ/kg, $v_{r3} = 2.175$

State 4: For the isentropic expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}\cdot\frac{v_2}{v_3} = \frac{v_1}{v_2}\cdot\frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}}\cdot\frac{T_2}{T_3} = \frac{621.2}{26.793}\cdot\frac{979.6}{2150} = 10.56$$

$$v_{r4} = \frac{v_4}{v_3}\cdot v_{r3} = 22.98 \Rightarrow T_4 = 1031\text{ K},\ u_4 = 785.75\text{ kJ/kg}$$

(a) The compression ratio is

$$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = \frac{621.2}{26.793} = 23.19 \leftarrow r$$

(b) The cutoff ratio is

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2150}{979.6} = 2.19 \leftarrow r_c$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2}$$

$$= \frac{(2440.3 - 1022.82) - (785.75 - 214.07)}{(2440.3 - 1022.82)}$$

$$= \frac{845.80}{1417.48} = 0.597\ (59.7\%) \leftarrow \eta$$

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300\ K)}{(95\ kPa)}\left|\frac{1\ kPa}{10^3 N/m^2}\right|\left|\frac{10^3 N\cdot m}{1\ kJ}\right|$$

$$= 0.9063\ m^3/kg$$

Thus

$$mep = \frac{(845.80\ kJ/kg)}{\left(0.9063\ \frac{m^3}{kg}\right)\left(1 - 1/23.19\right)}\left|\frac{10^3 N\cdot m}{1 kJ}\right|\left|\frac{1\ kPa}{10^3 N/m^2}\right|$$

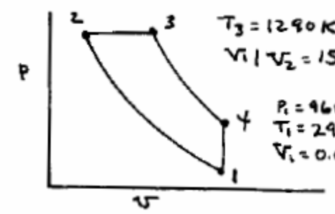
$$= 975\ kPa \longleftarrow mep$$

9.20 At the beginning of compression in an air-standard Diesel cycle, p_1 =96 kPa, V_1 =0.016 m^3, and T_1=290 K. The compression ratio is 15 and the maximum cycle temperature is 1290 K. Determine (a) the mass of air, in kg (b) the heat addition and heat rejection per cycle, each in kJ. (c) the net work, in kJ, and the thermal efficiency.

KNOWN: Operating data are provided for an air-standard Diesel cycle.

FIND: Determine (a) the mass of air, (b) the heat addition and heat rejection per cycle, (c) the net work and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$T_3 = 1290K$

$V_1/V_2 = 15$

$p_1 = 96 kPa$

$T_1 = 290K$

$V_1 = 0.016 m^3$

ASSUMPTIONS:

See Example 9.2

ANALYSIS: The mass m is

$$m = \frac{p_1 V_1}{RT_1} = \frac{(96\times10^3 N/m^2)(0.016 m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(290K)}$$

$$= 1.846\times10^{-2} kg \longleftarrow m$$

Next, fix each of the principal states: state 1: at $T_1 = 290K$, Table A-22 gives $u_1 = 206.91$ kJ/kg, $v_{r1} = 676.1$. State 2: for the isentropic compression

$v_{r2} = \frac{V_2}{V_1} v_{r1} = \left(\frac{676.1}{15}\right) = 45.073 \rightarrow T_2 = 818.6K$, $h_2 = 842.4\ \frac{kJ}{kg}$. State 3: with $T_3 = 1290K$, $h_3 = 1384.11$ kJ/kg, $v_{r3} = 11.555$. State 4: For the isentropic expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \left(\frac{V_4}{V_2}\right)\underbrace{\left(\frac{V_2}{V_3}\right)}_{=\frac{T_2}{T_3}\text{ (since pressure is constant)}} v_{r3} = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right)v_{r3} = (15)\left(\frac{818.6}{1290}\right)(11.555) = 109.99$$

($V_4 = V_1$)

Thus, $u_4 = 428.12$ kJ/kg.

The heat addition is $Q_{23} = m(h_3 - h_2) = (1.846\times10^{-2})(1384.11 - 842.4) = 10 kJ \longleftarrow Q_{23}$

The heat rejection is $Q_{41} = m(u_4 - u_1) = (1.846\times10^{-2})(428.12 - 206.91) = 4.084$ kJ $\longleftarrow Q_{41}$

For any cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle} = Q_{23} - Q_{41} = 10 kJ - 4.084 kJ = 5.916 kJ \longleftarrow W_{cycle}$$

The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{5.916}{10} = .592\ (59.2\%) \longleftarrow \eta$$

9.30 Consider an ideal air-standard Brayton cycle with minimum and maximum temperatures of 300 K and 1500 K, respectively. The pressure ratio is that which maximizes the net work developed by the cycle per unit mass of air flow. On a cold air-standard basis, calculate

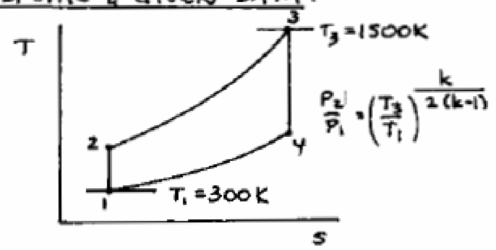
(a) the compressor and turbine work per unit mass of air flow, each in kJ/kg.

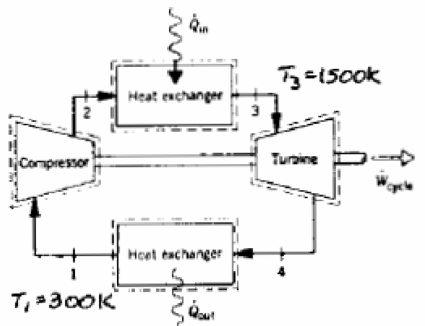
(b) the thermal efficiency of the cycle.

KNOWN: An ideal cold air-standard Brayton cycle has known minimum and maximum temperatures a pressure that corresponds to the maximum net work per unit mass of air flow.

FIND: Determine (a) the compressor and turbine work per unit of mass flow, (b) thermal efficiency. (c) Plot the thermal efficiency versus the maximum cycle temperature ranging from 1200 to 1800 K.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.5.

ANALYSIS: First, determine the unknown temperatures at principal states 2 and 4. From the solution to Example 9.5

$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = \left(\frac{1500}{300}\right)^{\frac{1.4}{2(1.4-1)}} = 16.719$$

where $k = 1.4$ from Table A-20. For the isentropic compression

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} T_1 = 670.8\ ^\circ R$$

and for the iseutropic expansion

$$T_4 = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} T_3 = 670.8\ ^\circ R$$

(a) The compressor work per unit mass of air flow is

$$\frac{\dot{W}_{comp}}{\dot{m}} = h_2 - h_1 = c_p(T_2 - T_1)$$

$$= (1.005\ kJ/kg\cdot K)(670.8 - 300)K = 372.6\ \frac{kJ}{kg} \longleftarrow \dot{W}_{comp}/\dot{m}$$

Similarly, for the turbine

$$\frac{\dot{W}_{turb}}{\dot{m}} = c_p(T_3 - T_4)$$

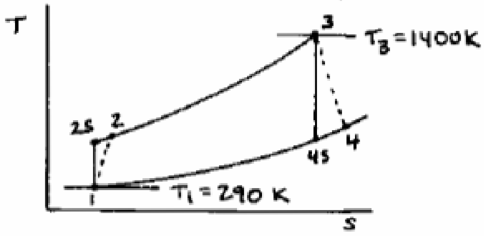
$$= (1.005)(1500 - 670.8) = 833.3\ kJ/kg \longleftarrow \dot{W}_{turb}/\dot{m}$$

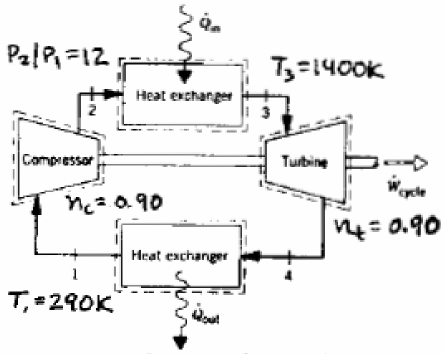
9.34 The compressor and turbine of a simple gas turbine each have isentropic efficiencies of 90%. The compressor pressure ratio is 12. The minimum and maximum temperatures are 290 K and 1400 K, respectively. On the basis of an air-standard analysis, compare the values of (a) the net work per unit mass of air flowing, in kJ/kg, (b) the heat rejected per unit mass of air flowing, in kJ/kg, and (c) the thermal efficiency to the same quantities evaluated for an ideal cycle.

KNOWN: An air-standard gas turbine cycle has a known compressor pressure ratio and specified minimum and maximum temperatures. The compressor and turbine each have isentropic efficiencies of 82%.

FIND: Determine (a) the net work per unit mass of air flow, (b) the heat rejected per unit mass of air flow, and (c) the thermal efficiency and compare them to the same quantities evaluated for an ideal cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.6

ANALYSIS: First, fix each of the principal states for each cycle using data from Table A-22.

State 1 $T_1 = 290K \Rightarrow h_1 = 290.16$ kJ/kg , $P_{r1} = 1.2311$

State 2 First, determine h_{2s}. For isentropic compression

$$P_{r2s} = (P_2/P_1)\,P_{r1} = 14.7732 \Rightarrow h_{2s} = 590.47 \text{ kJ/kg}$$

Using the compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$;

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 623.84 \text{ kJ/kg}$$

State 3 $T_3 = 1400K \Rightarrow h_3 = 1515.42$ kJ/kg , $P_{r3} = 450.5$

State 4 First, determine h_{4s}. For isentropic expansion

$$P_{r4s} = (P_4/P_3)\,P_{r3} = 37.542 \Rightarrow h_{4s} = 768.38 \text{ kJ/kg}$$

Using the turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$;

$$h_4 = h_3 - \eta_t\,(h_3 - h_{4s}) = 843.08 \text{ kJ/kg}$$

(a) The net work per unit mass of air flowing is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1)$$
$$= (1515.4 - 843.08) - (623.84 - 290.16)$$
$$= 338.64 \text{ kJ/kg}$$

$\dot{W}_{cycle}/\dot{m}$

For the ideal cycle

$$\left(\frac{\dot{W}_{cycle}}{\dot{m}}\right)_{ideal} = (h_3 - h_{4s}) - (h_{2s} - h_1)$$
$$= 446.7 \text{ kJ/kg}$$

(ideal cycle)

Thus, irreversibilities reduce the net work significantly.

(b) The heat rejected per unit mass of air flowing is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1 = 843.08 - 290.16 = 552.92 \text{ kJ/kg} \longleftarrow \quad \dot{Q}_{out}/\dot{m}$$

For the ideal cycle

$$\left(\frac{\dot{Q}_{out}}{\dot{m}}\right)_{ideal} = h_{4s} - h_1 = 478.2 \text{ kJ/kg} \longleftarrow \quad \text{(ideal cycle)}$$

Thus, less heat is rejected for the ideal cycle.

(c) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{552.92}{(1515.4 - 623.84)}$$

$$= 0.38 \ (38\%) \longleftarrow \quad \eta$$

For the ideal cycle

$$\eta = 1 - \frac{h_{4s} - h_1}{h_3 - h_{2s}} = 0.483 \ (48.3\%) \longleftarrow \quad \text{(ideal cycle)}$$

Thus, irreversibilities cause a substantial decrease in thermal efficiency.

9.39 A regenerative gas turbine power plant is shown in Fig. P9.39. Air enters the compressor at 1 bar, 27 °C with a mass flow rate of 0.562 kg/s and is compressed to 4 bar. The isentropic efficiency of the compressor is 80%, and the regenerator effectiveness is 90%. All the power developed by the high-pressure turbine is used to run the compressor. The low-pressure turbine provides the net power output. Each turbine has an isentropic efficiency of 87% and the temperature at the inlet to the highpressure turbine is 1200 K. Determine

(a) the net power output, in kW.

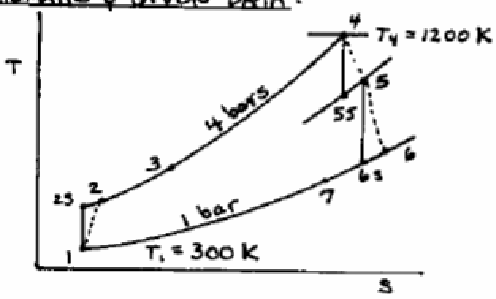
(b) the thermal efficiency.

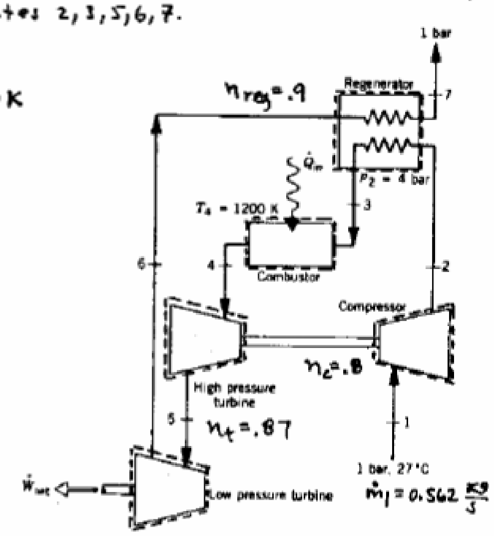
(c) the temperature of the air at states 2, 3, 5, 6, and 7, in K.

KNOWN: In a regenerative gas turbine power plant, a high pressure turbine runs the compressor and the net power output is provided by a low pressure turbine. Data are known at various locations.

FIND: Determine (a) the net power output, (b) the thermal efficiency, and (c) the temperature at states 2, 3, 5, 6, 7.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressor, turbines, and regenerator are adiabatic. (3) There are no pressure drops for flow through the heat exchangers. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1: $T_1 = 300K \Rightarrow h_1 = 300.19$ kJ/kg, $Pr_1 = 1.3860$

State 2: For an isentropic compression, $Pr_{2s} = (P_2/P_1)\,Pr_1 = 5.544 \Rightarrow h_{2s} = 446.49\ \frac{kJ}{kg}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s}-h_1)}{\eta_c} = 483.06\ kJ/kg,\ T_2 = 481\,K$$

State 4: $T_4 = 1200K \Rightarrow h_4 = 1277.79$ kJ/kg

State 5: The work of the compressor and the high pressure turbine are equal. Thus; $h_2 - h_1 = h_4 - h_5 \Rightarrow h_5 = 1094.92$ kJ/kg
From the turbine efficiency: $h_4 - h_5 = \eta_t\,(h_4 - h_{5s}) \Rightarrow h_{5s} = 1067.59$
Thus, $Pr_{5s} = 122.86$ and $p_5 = (Pr_{5s}/Pr_4)\,P_4 = 2.065$ bars
From $h_5 = 1094.92 \Rightarrow Pr_5 = 134.714$, $T_5 = 1043K$

State 6: For isentropic expansion through the low pressure turbine
$Pr_{6s} = (P_6/P_5)\,Pr_5 = (1/2.065)(134.714) = 65.237 \Rightarrow h_{6s} = 896.40$
Using the turbine efficiency, $h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 922.21$ kJ/kg, $T_6 = 890K$

State 3: Now, using the regenerator effectiveness

$$\eta_{reg} = \frac{h_3-h_2}{h_6-h_2} \Rightarrow h_3 = \eta_{reg}(h_6-h_2) + h_2 = 878.3\ kJ/kg,\ T_3 = 851\,K$$

State 7: Applying mass and energy rate balances to a control volume enclosing the regenerator, $0 = (h_2-h_3) + (h_6-h_7)$ or
$h_7 = (h_2-h_3) + h_6 = 483.06 - 878.3 + 922.21 = 526.97$ kJ/kg,
$T_7 = 523\,K$

The net power output is the power developed by the low pressure turbine:

$$\dot{W}_{net} = \dot{m}\,(h_5 - h_6)$$
$$= \left(0.562\ \frac{kg}{s}\right)(1094.92 - 922.21)\ \frac{kJ}{kg}\ \left|\frac{1\ kW}{1\ kJ/s}\right|$$
$$= 97.1\ kW$$

The thermal efficiency is

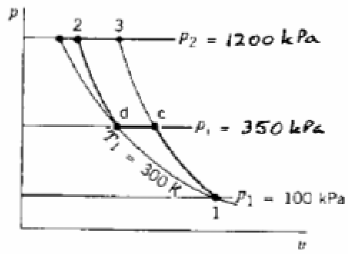
$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}\,(h_4-h_3)}$$
$$= \frac{(97.1\ kJ/s)}{\left(0.562\ \frac{kg}{s}\right)[1277.79 - 878.3]\ \frac{kJ}{kg}} = 0.432\ (43.2\%)$$

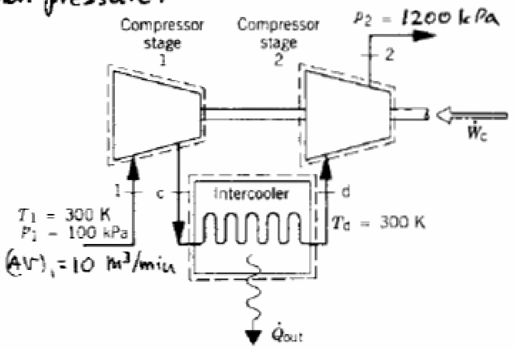
9.45 A two-stage air compressor operates at steady state, compressing 10 m^3/min of air from 100 kPa, 300 K, to 1200 kPa. An intercooler between the two stages cools the air to 300 K at a constant pressure of 350 kPa. The compression processes are isentropic. Calculate the power required to run the compressor, in kW, and compare the result to the power required for isentropic compression from the same inlet state to the same final pressure.

KNOWN: Air is compressed in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are given.

FIND: Determine the power to run the compressor and compare this to the power required for isentropic compression from the same inlet state to the same final pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.9.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1: $T_1 = 300\text{K} \Rightarrow h_1 = 300.19 \text{ kJ/kg}, \; p_{r1} = 1.3860$

State c: $p_{rc} = (p_c/p_1)\, p_{r1} = 4.851 \Rightarrow h_c = 429.77 \text{ kJ/kg}$

State d: $T_d = T_1 = 300\text{K} \Rightarrow h_d = 300.19 \text{ kJ/kg}, \; p_{rd} = 1.3860$

State 2: $p_{r2} = (p_2/p_d)\, p_{rd} = 4.752 \Rightarrow h_2 = 427.21 \text{ kJ/kg}$

The mass flow rate is

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1} = \frac{(10\,m^3/\text{min})(100\text{kPa})}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{10^3\,N/m^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\,N\cdot m}\right| = 0.1936 \text{ kg/s}$$

The compressor power is

$$\dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = \dot{m}(h_c - h_1) + \dot{m}(h_2 - h_d)$$

$$= (0.1936 \tfrac{kg}{s})(429.77 - 300.19)\tfrac{kJ}{kg}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| + (0.1936)(427.21 - 300.19)$$

$$= 25.087 \text{ kW} + 24.591 \text{ kW} = 49.68 \text{ kW} \longleftarrow \dot{W}_c$$

To find the power a single stage of compression, determine h_3 as follows:

$$p_{r3} = (p_2/p_1)\, p_{r1} = 16.632 \Rightarrow h_3 = 610.65 \text{ kJ/kg}$$

Thus $\quad \dot{W}_c = \dot{m}(h_3 - h_1) = 60.11 \text{ kW}$

The decrease in power with two-stage, intercooled compression is

$$\%\text{ decrease} = \frac{60.11 - 49.68}{60.11} \times 100 = 17.35\% \longleftarrow \text{\% reduction in power}$$