Lecture 38:
Vapor-compression refrigeration systems

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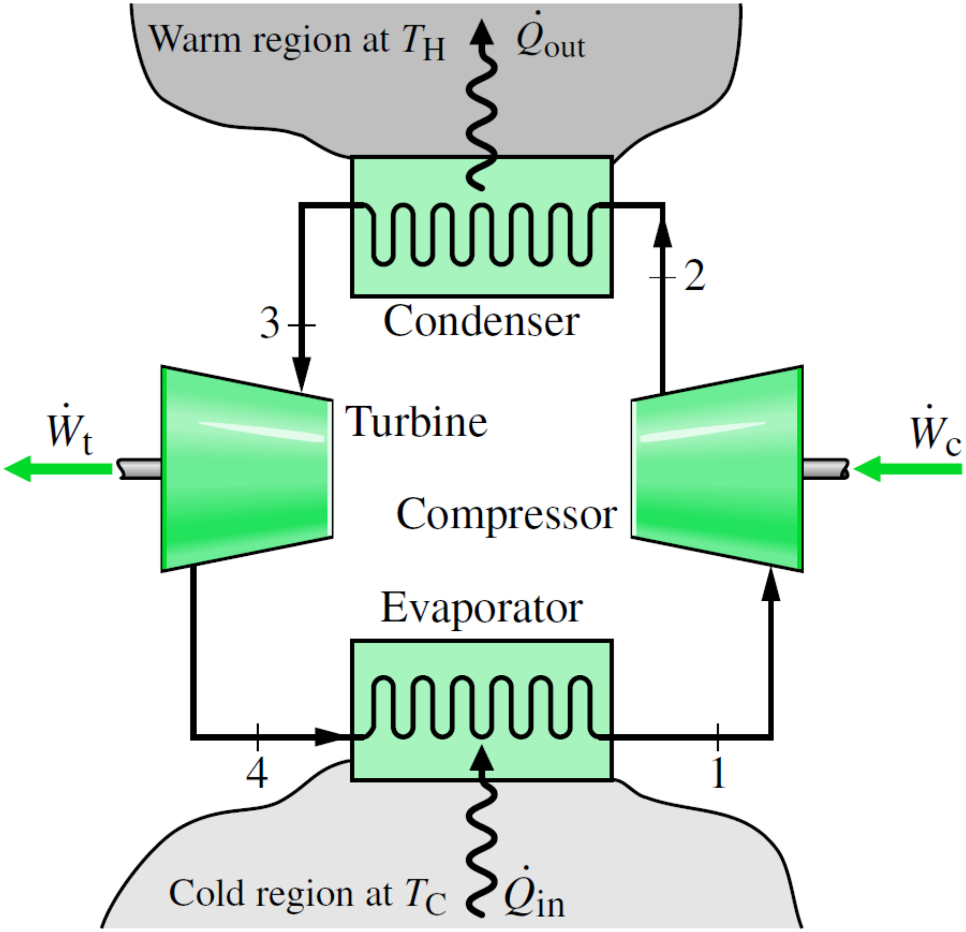
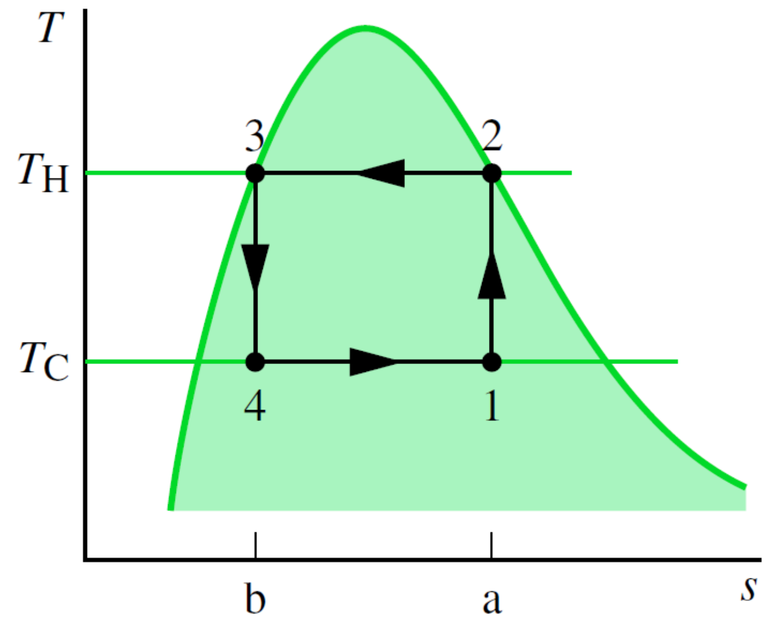
Carnot Refrigeration Cycle





Coefficient of Performance β

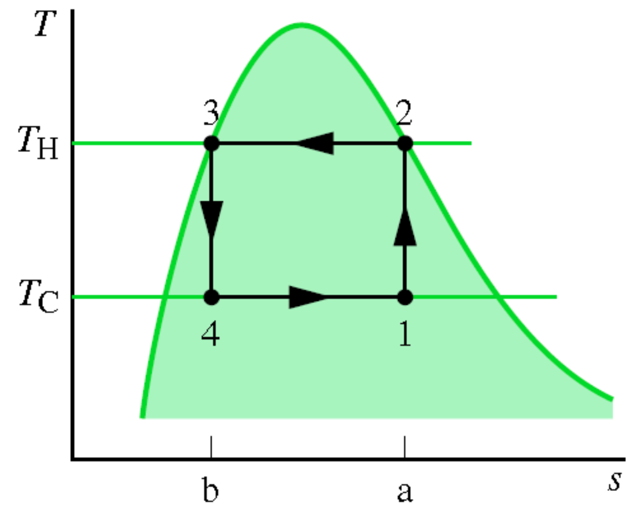
- Coefficient Of Performance ::: Ratio of the refrigeration effect to the net work input required to achieve that effect.

(COP)

$$\boxed{\beta_{\max} = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_{\text{c}}/\dot{m} - \dot{W}_{\text{t}}/\dot{m}}}$$

$$= \frac{\text{area 1–a–b–4–1}}{\text{area 1–2–3–4–1}} = \frac{T_{\text{C}}(s_{\text{a}} - s_{\text{b}})}{(T_{\text{H}} - T_{\text{C}})(s_{\text{a}} - s_{\text{b}})}$$

$$= \frac{T_{\text{C}}}{T_{\text{H}} - T_{\text{C}}}$$



Represents the maximum theoretical coefficient of performance of any refrigeration cycle operating between regions at T_C and T_H.

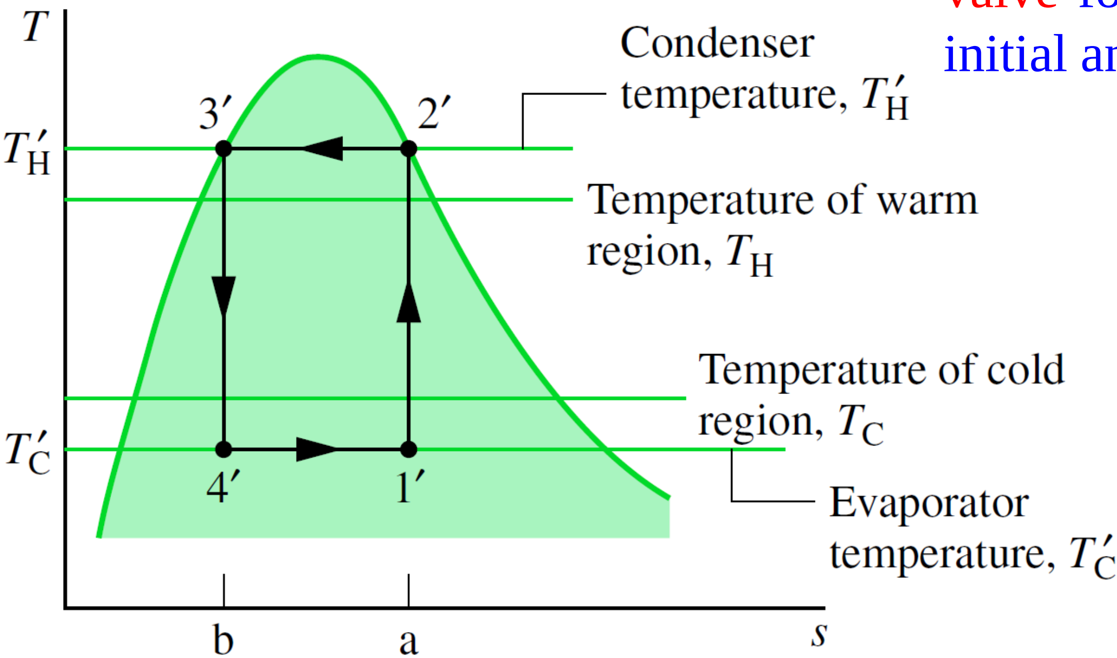
Departures from the Carnot Cycle

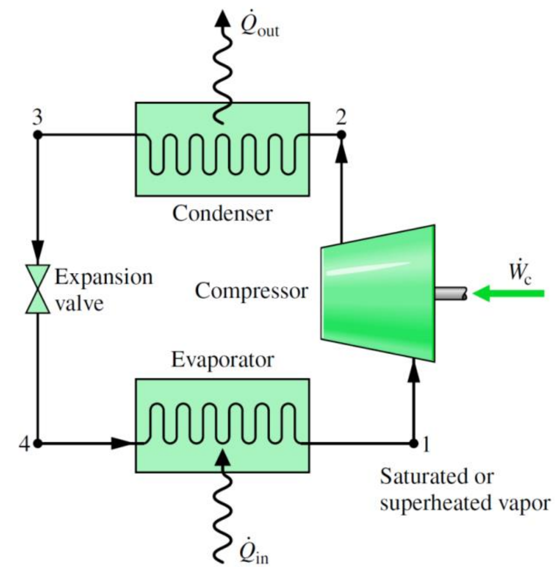
Temperature difference ΔT $\beta' = \dfrac{\text{area } 1'\text{–a–b–}4'\text{–}1}{\text{area } 1'\text{–}2'\text{–}3'\text{–}4'\text{–}1'} = \dfrac{T'_C}{T'_H - T'_C}$

.Compression two-phase liquid–vapor mixture. Wet compression.

In actual systems, compressor handles vapor only. *Dry compression.*

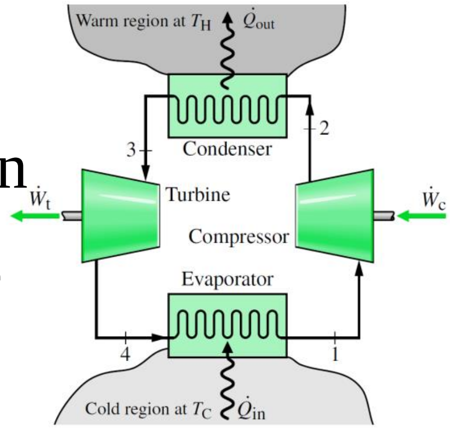
The work output of the turbine is normally sacrificed by using a simple throttling valve for the expansion turbine, saving in initial and maintenance costs.

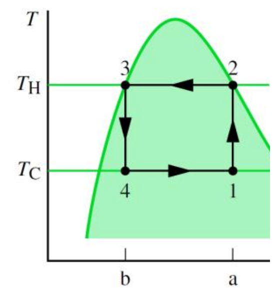


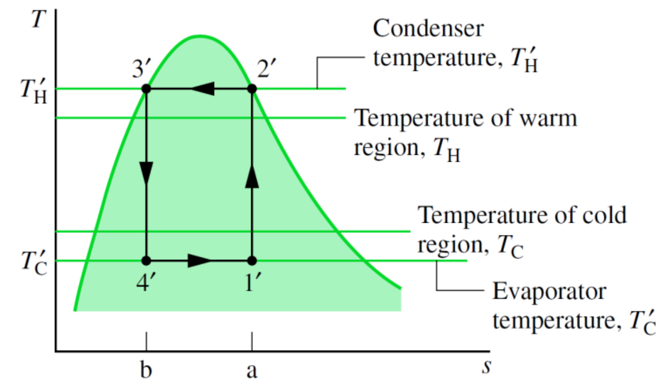


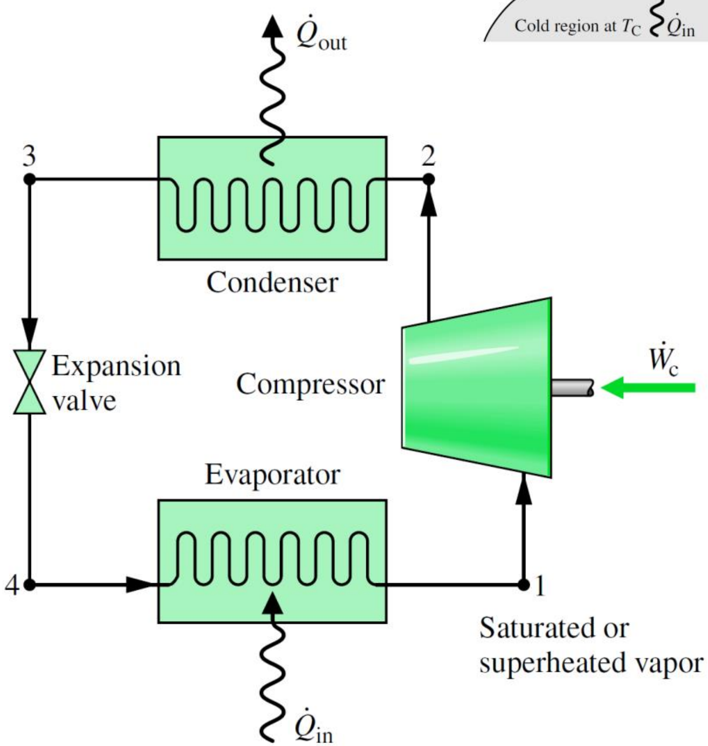
Departures from the Carnot Cycle

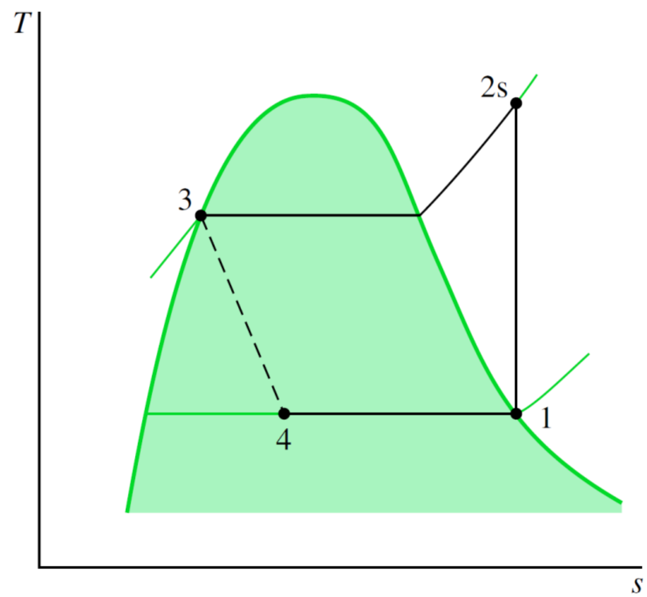
- ΔT
- Dry compression
- Throttling valve



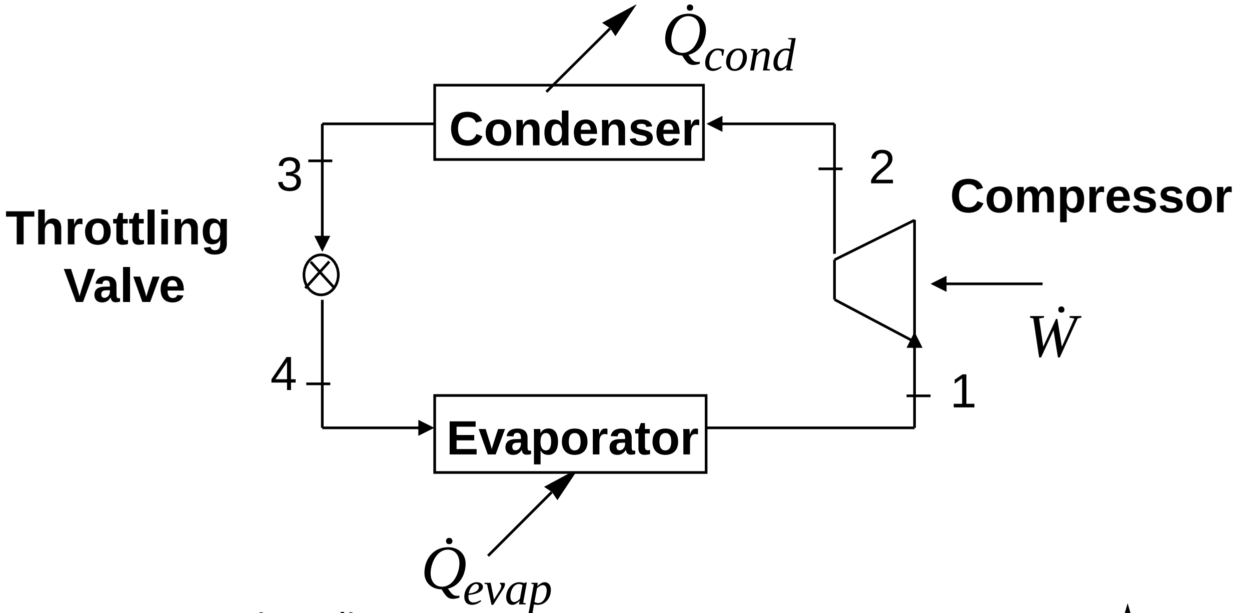


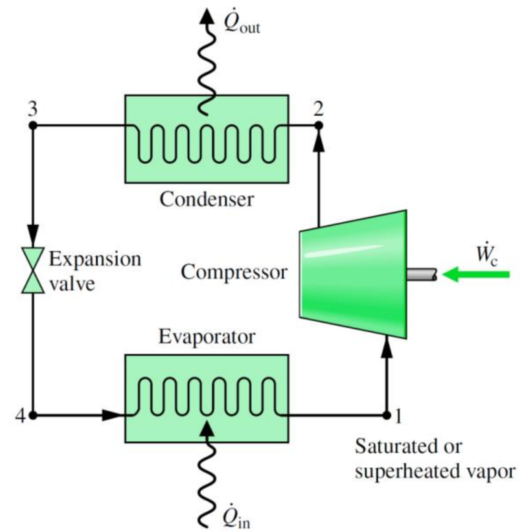


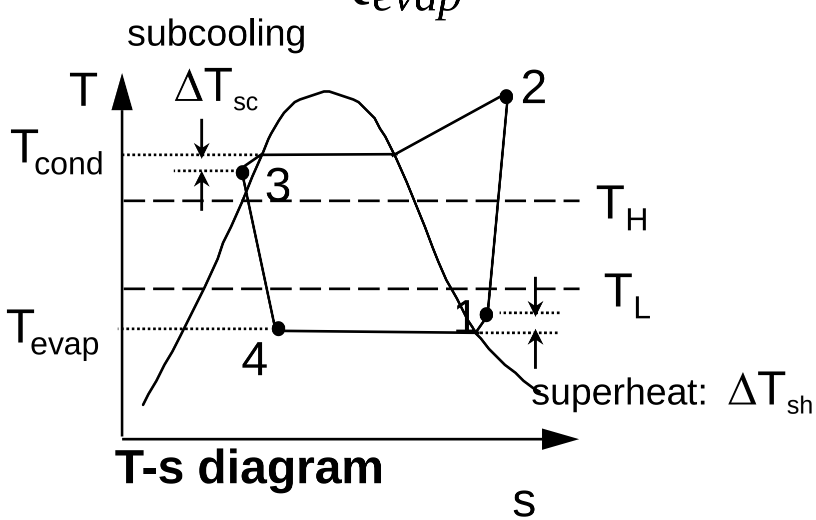




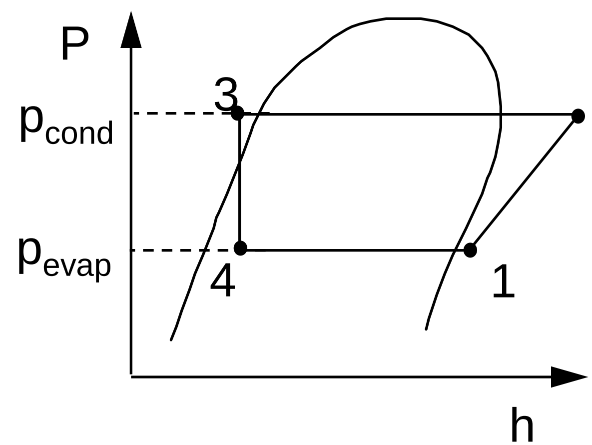
Cycle Analysis





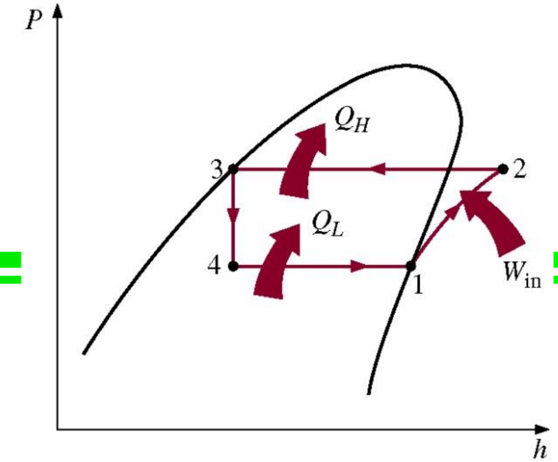


T-s diagram



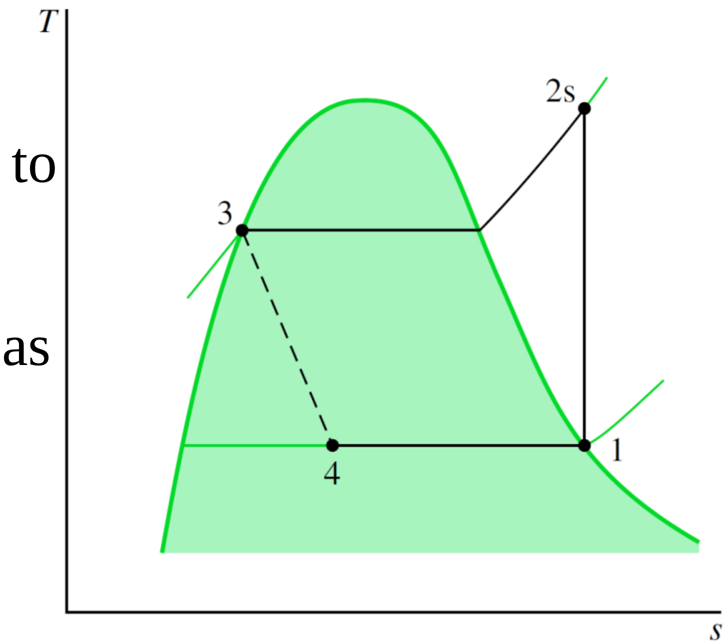
P-h diagram

Ideal Vapor-Compression Refrigeration Cycle

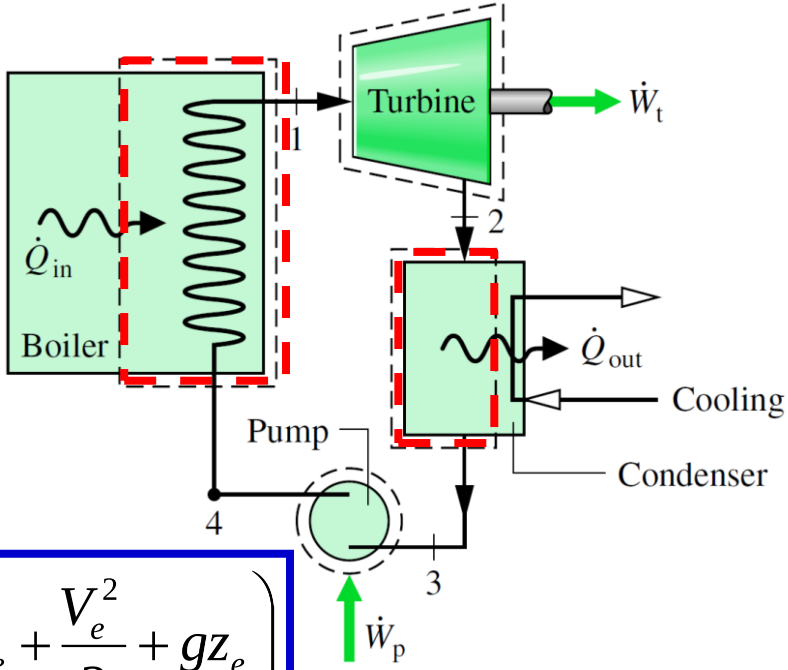


- Process 1–2s: **Isentropic compression** of the refrigerant from state 1 to the condenser pressure at state 2s.
- Process 2s–3: Heat transfer from the refrigerant as it flows at **constant pressure** through the condenser. The refrigerant exits as a liquid at state 3.
- Process 3–4: Throttling process from state 3 to a **two-phase liquid–vapor mixture** at 4.
- Process 4–1: Heat transfer to the refrigerant as it flows at **constant pressure** through the evaporator to complete the cycle.

All of the processes in the above cycle are internally reversible except for the throttling process.



Analyzing Rankine Cycle---I



- **Assumptions:** 1) Steady state. 2) ΔKE=ΔPE=0. 3) stray Q=0
- Turbine

$$\boxed{\frac{dE_{cv}}{dt}=\dot{Q}_{cv}-\dot{W}_{cv}+\sum_i \dot{m}_i\left(h_i+\frac{V_i^2}{2}+gz_i\right)-\sum_e \dot{m}_e\left(h_e+\frac{V_e^2}{2}+gz_e\right)}$$

$$0=\cancelto{0}{\dot{Q}_{cv}}-\dot{W}_t+\dot{m}\left[h_1-h_2+\cancelto{0}{\frac{V_1^2-V_2^2}{2}}+\cancelto{0}{g(z_1-z_2)}\right]$$

$$\boxed{\frac{\dot{W}_t}{\dot{m}}=h_1-h_2}$$

- Condenser $\boxed{\frac{\dot{Q}_{out}}{\dot{m}}=h_2-h_3}$
- Pump $\boxed{\frac{\dot{W}_p}{\dot{m}}=h_4-h_3}$
- Boiler $\boxed{\frac{\dot{Q}_{in}}{\dot{m}}=h_1-h_4}$

Vapor-compression refrigeration cycle

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$$0 = \cancelto{0}{\dot{Q}_{\mathrm{cv}}} - \dot{W}_{\mathrm{t}} + \dot{m} \left[h_1 - h_2 + \cancelto{0}{\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)} \right]$$

- **Evaporator:**

$$\boxed{\frac{\dot{Q}_{\mathrm{in}}}{\dot{m}} = h_1 - h_4}$$

- The heat transfer rate is referred to as the refrigeration capacity. (kW).
  - » Another unit for the refrigeration capacity is the ton of refrigeration, = 211 kJ/min.

- **Compressor**

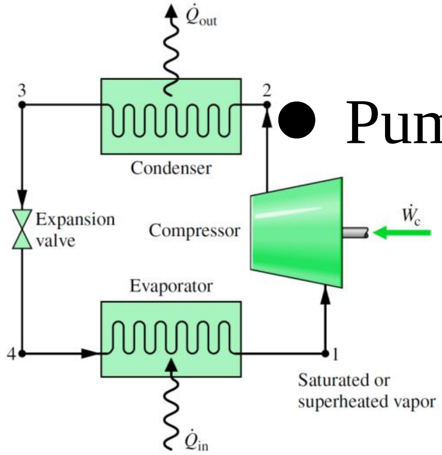
$$\boxed{\frac{\dot{W}_{\mathrm{c}}}{\dot{m}} = h_2 - h_1}$$

- **Condenser**

$$\boxed{\frac{\dot{Q}_{\mathrm{out}}}{\dot{m}} = h_2 - h_3}$$

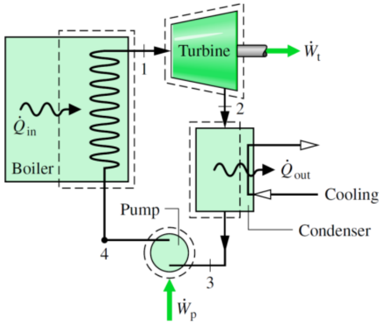
- **Throttling process**

$$\boxed{h_4 = h_3}$$



- Boiler

$$\boxed{\frac{\dot{Q}_{\mathrm{in}}}{\dot{m}} = h_1 - h_4}$$



- Turbine

$$\boxed{\frac{\dot{W}_{\mathrm{t}}}{\dot{m}} = h_1 - h_2}$$

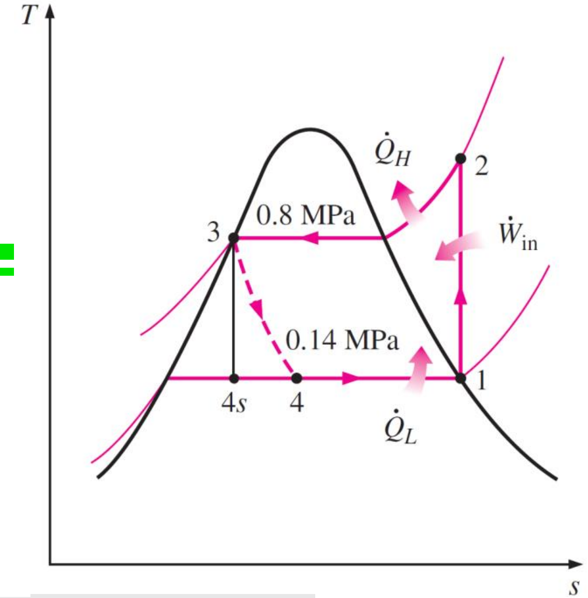
- Condenser

$$\boxed{\frac{\dot{Q}_{\mathrm{out}}}{\dot{m}} = h_2 - h_3}$$

- Pump

$$\boxed{\frac{\dot{W}_{\mathrm{p}}}{\dot{m}} = h_4 - h_3}$$

Example 1: Ideal Vapor-Compression Refrigeration Cycle



- **Known:** R-134a as the working fluid and operates on an ideal vapor-compression refrigeration cycle between 0.14 and 0.8 MPa. mass flow rate of the refrigerant is 0.05 kg/s,
- **Find:** Q_{in}, W_{in}, mass flow rate, Q_{out}
- **Assumptions:** 1) Steady state. 2) ΔKE=ΔPE=0.
- **Analysis:**

$$P_1 = 0.14 \text{ MPa} \longrightarrow$$

$$h_1 = h_{g\,@\,0.14\text{ MPa}} = 239.16 \text{ kJ/kg}$$

$$s_1 = s_{g\,@\,0.14\text{ MPa}} = 0.94456 \text{ kJ/kg}\cdot\text{K}$$

$$\left.\begin{array}{l} P_2 = 0.8 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} \quad h_2 = 275.39 \text{ kJ/kg}$$

$$P_3 = 0.8 \text{ MPa} \longrightarrow$$

$$h_3 = h_{f\,@\,0.8\text{ MPa}} = 95.47 \text{ kJ/kg}$$

$$h_4 \cong h_3 \text{ (throttling)} \longrightarrow$$

$$h_4 = 95.47 \text{ kJ/kg}$$

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = \mathbf{7.18\ kW}$$

$$= (0.05 \text{ kg/s})[(239.16 - 95.47) \text{ kJ/kg}]$$

$$\dot{W}_{in} = \dot{m}(h_2] = \mathbf{1.81\ kW}$$

$$= (0.05 \text{ kg/s})[(275.39 - 239.16) \text{ kJ/kg}]$$

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.05 \text{ kg/s})[(275.39 - 95.47) \text{ kJ/kg}] = \mathbf{9.0\ kW}$$

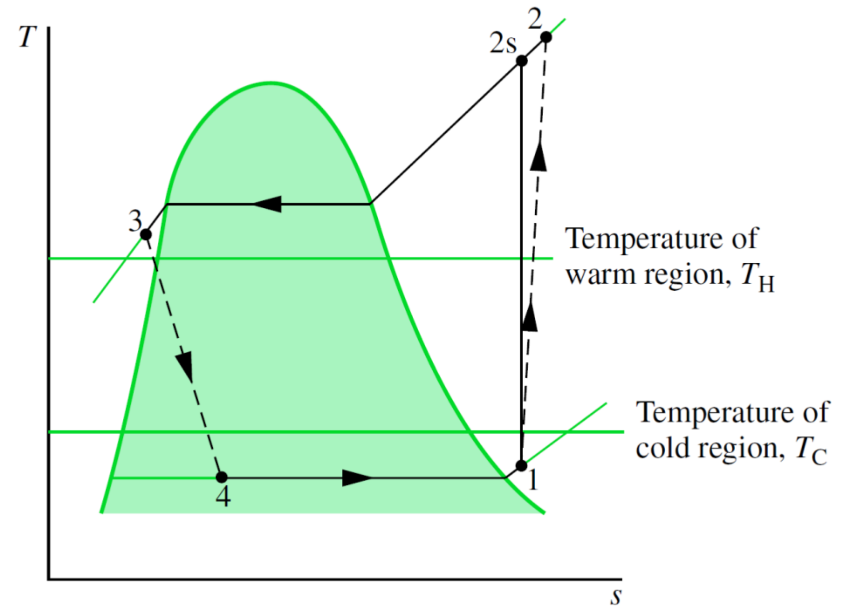
$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 7.18 + 1.81 = 8.99 \text{ kW}$$

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.18 \text{ kW}}{1.81 \text{ kW}} = \mathbf{3.97}$$

Actual cycle

- The heat transfers between the refrigerant and the warm and cold regions are not accomplished reversibly: $T_{evaporator} < T_C$, $T_{condenser} > T_H$.
  - » The COP $\downarrow$ as $T_{evaporator}\downarrow$ and COP $\downarrow$ as $T_H\uparrow$.
- Adiabatic irreversible compression

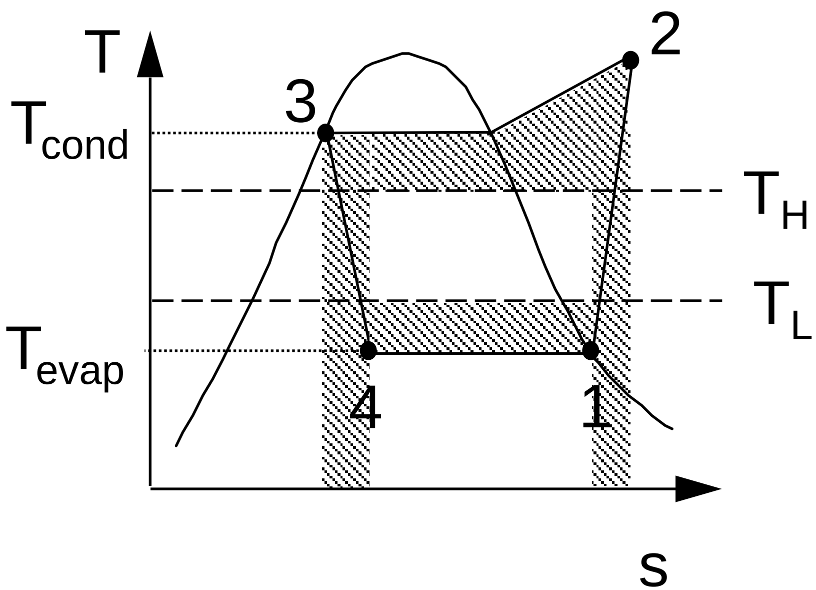
$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$



Cycle Analysis

Second Law Efficiency: $$\varepsilon = \frac{COP_{actual}}{COP_{Carnot}}$$

Irreversibilities

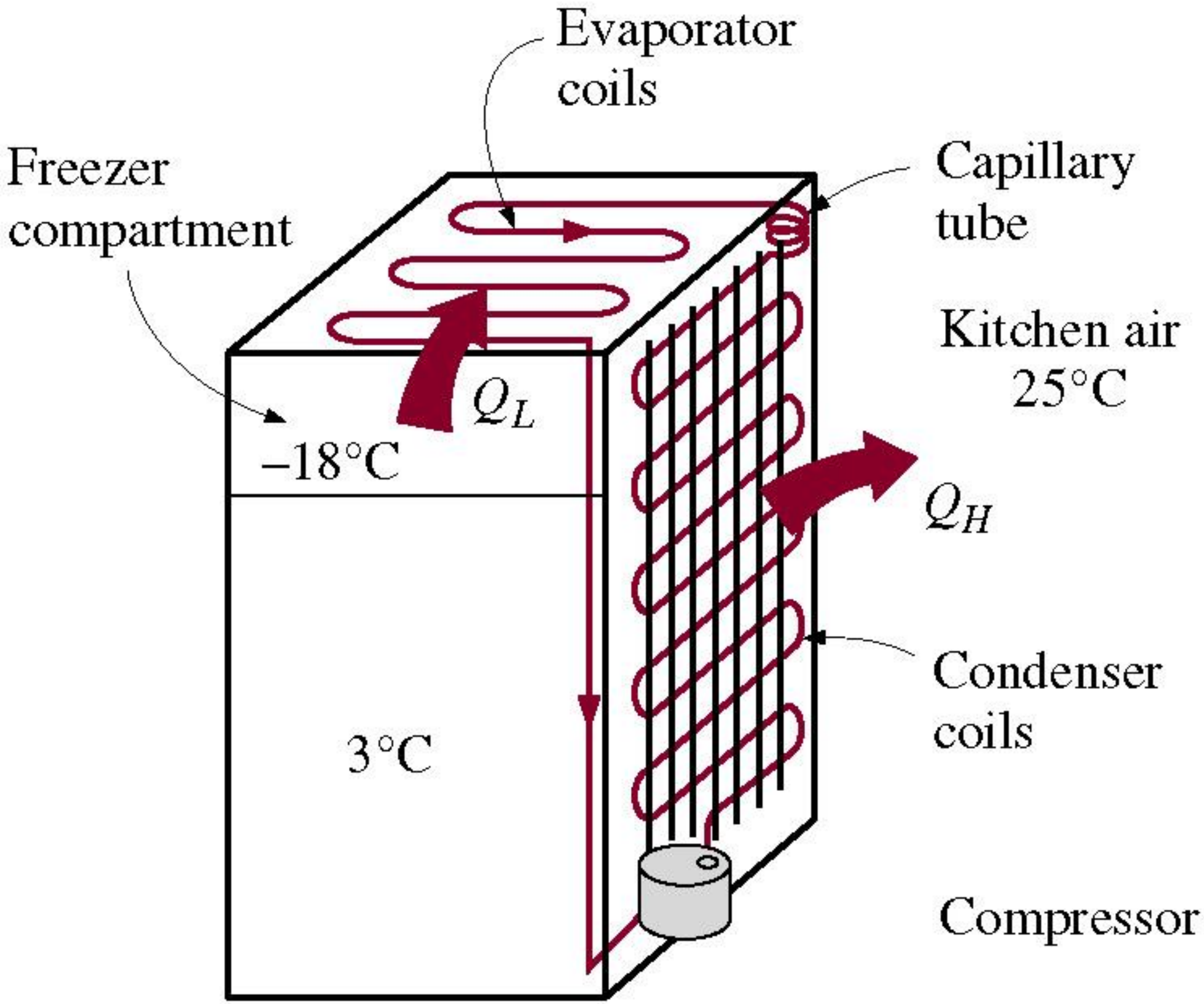


Compressor Analysis

- **Overall isentropic Efficiency:::** Ratio of isentropic compressor power input to actual compressor power input:

$$\eta_{o,is} = \frac{\dot{m}_r \left(h_{2s} - h_1\right)}{\dot{W}_{comp}}$$

Household Refrigerator



Vapor Compression Cycles

- Best possible performance for a heat pump cycle:
  - » Carnot Cycle!

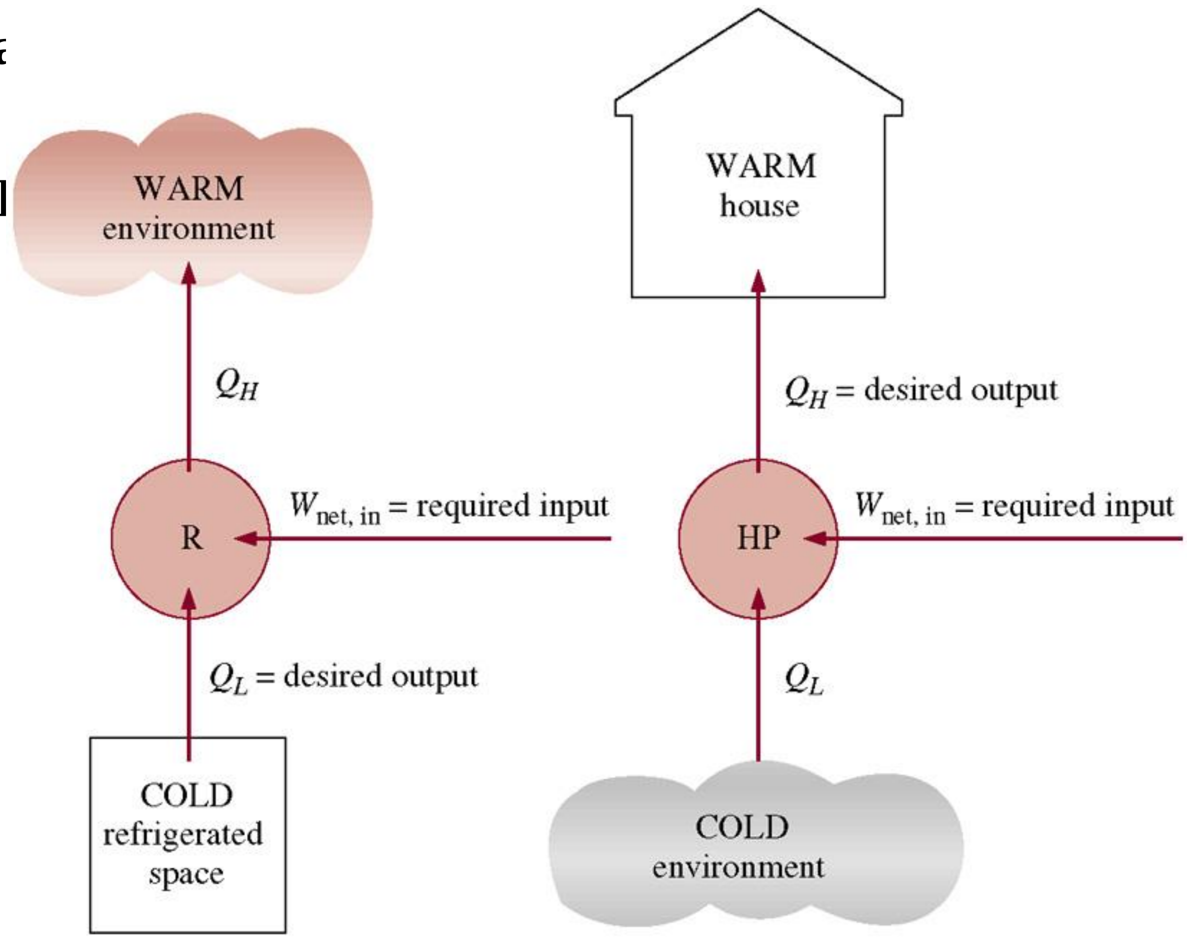
- What is the difference between

  a.) Refrigeration/Air Conditioning, and

  b.) Heat Pumping?

Refrigerator & Heat pump

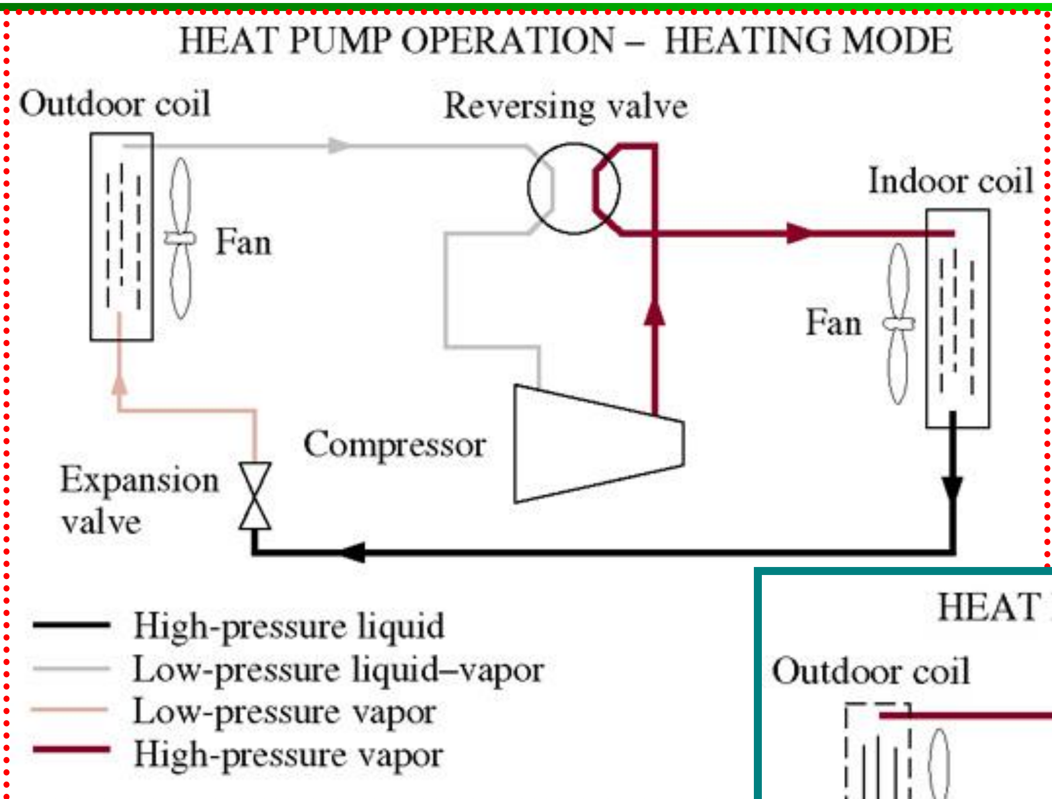
- **The objective of a refrigerator is to remove hea** ***(QL)*** **from the cold medium;**
- **The objective of a heat pum is to supply heat** ***(QH)*** **to a warm medium**

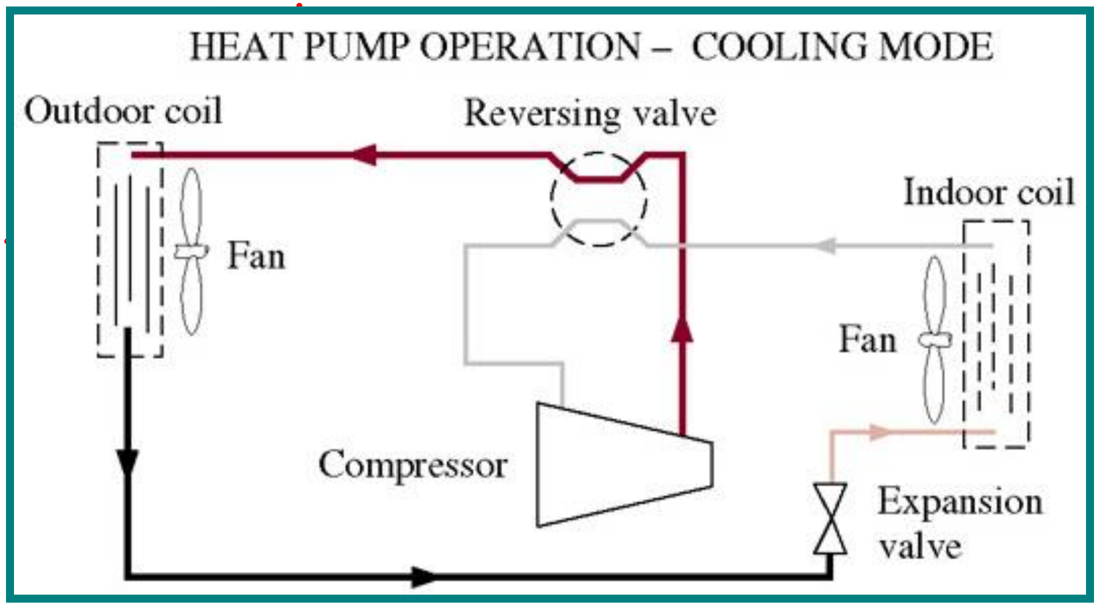


(*a*) Refrigerator

(*b*) Heat pump

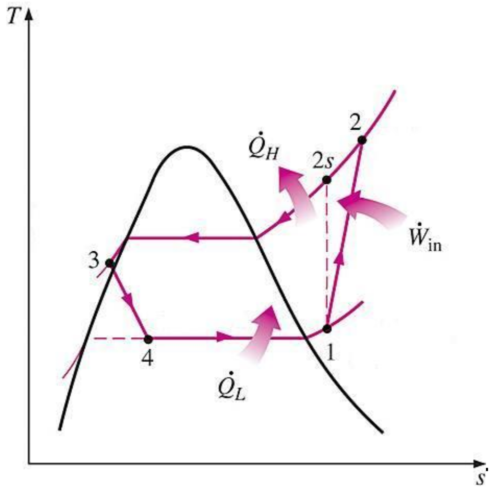
A heat pump can be used to heat a house in winter and to cool it in summer





Example 2:Vapor Compression Cycles

- Consider a 300 kJ/min refrigeration system that operates on an vapor-compression cycle with R134a as the working fluid. The refrigerant enters the compressor as superheated vapor at 140 kPa and is compressed to 800 kPa. If the isentropic efficiency of the compressor is 0.85, and the superheat and subcooling are 5 ^oC, determine: a) the quality of the refrigerant at the end of the throttling process, b) the coefficient of performance, c) the power input to the compressor (kW).
- Given: $p_1 = p_4 = 140$ kPa, $\Delta T_{superheat} = 5\ ^oC$, $p_2 = p_3 = 800$ kPa, $\Delta T_{supcooling} = 5\ ^oC$, $Q_L = 300$ kJ/min Find: x_4 , COP_R , W_{in}
- Assumptions:
  - » Steady state, steady flow, Neglect ΔKE and ΔPE
  - » Adiabatic compression
  - » Constant pressure heat transfer processes
  - » No work or heat interactions in the expansion process



Example 2

$p_1 = p_4 = 140$ kPa, $\Delta T_{superheat} = 5\ ^oC$, $p_2 = p_3 = 800$ kPa, $\Delta T_{supcooling} = 5\ ^oC$, $Q_L = 300$ kJ/min

- Solution:
  - » Quality at the end of the throttling process:

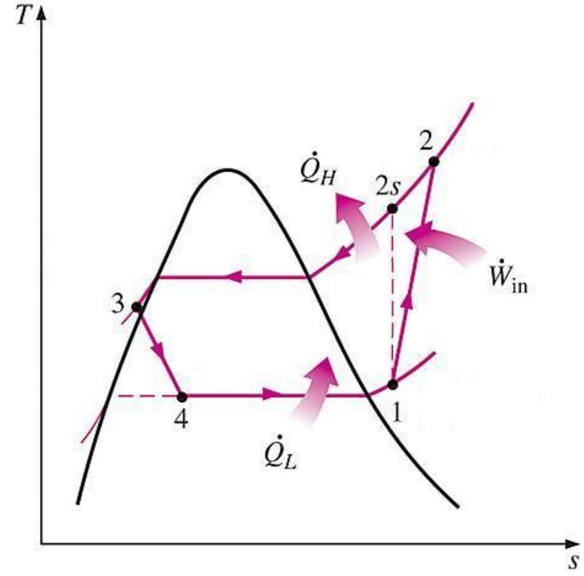
$$h_3 = h_4$$

$$x_4 = \frac{h_4 - h_f\left(p_4\right)}{h_{fg}\left(p_4\right)}$$

$$h_3 = h\left(p_3, T_3\right)$$

$$T_3 = T_{sat}\left(800kPa\right) - \Delta T_{subcooling}$$

$$T_3 = 26.31^oC$$



  - » From R134a Tables:

$h_3 \approx h_f(T_3) = 87.85$ kJ/kg

$h_f(p_4) = 27.08$ kJ/kg

$h_{fg}(p_4) = 212.08$ kJ/kg

$$\boxed{\Rightarrow x_4 = 0.287}$$

Example 2

$p_1 = p_4 = 140$ kPa, $\Delta T_{superheat} = 5\ ^oC$, $p_2 = p_3 = 800$ kPa, $\Delta T_{supcooling} = 5\ ^oC$, $Q_L = 300$ kJ/min

- Continue Solution:
  - » Coefficient of Performance cooling:

$$COP_R = \frac{h_1 - h_4}{h_2 - h_1}$$

$$h_1 = h(p_1, T_1)$$

$$T_1 = T_{sat}(p_1) + \Delta T_{\text{superheat}}$$

$$T_1 = -18.77\,^oC + 5\,^oC = -13.77\,^oC$$

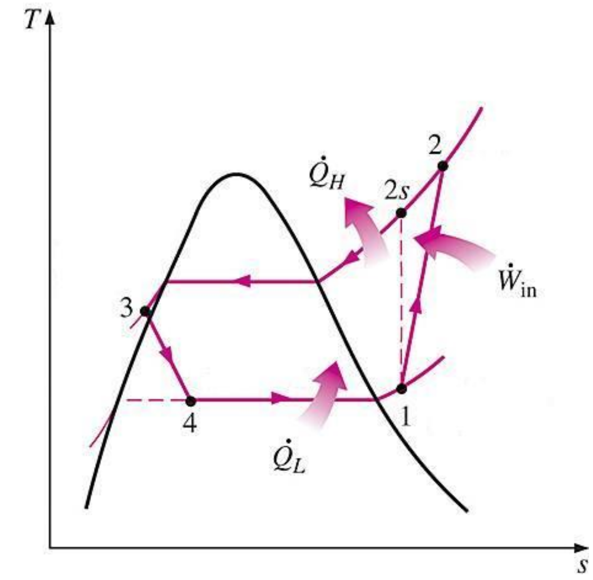
$$h_1 \approx 242.12\ {}^{kJ}/_{kg}$$

– Compressor analysis:

$$h_{2s} = h(p_2, s_{2s})$$

$$s_{2s} = s_1 \approx 0.9604\ {}^{kJ}/_{kg-K}$$

$$\Rightarrow h_{2s} \approx 280.41\ {}^{kJ}/_{kg-K}$$



Example 2

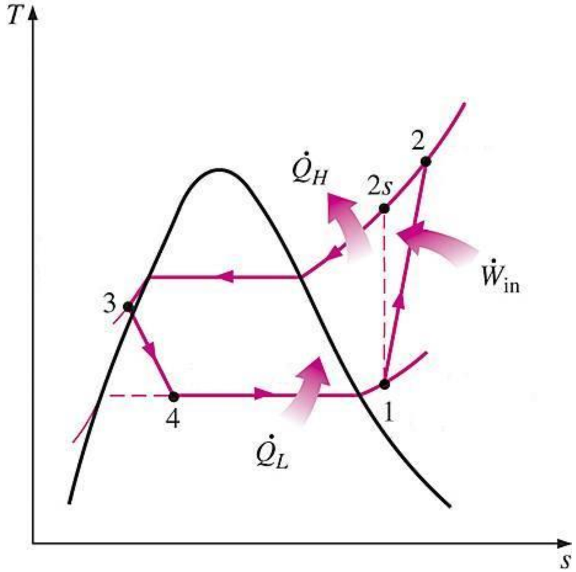
$p_1 = p_4 = 140$ kPa, $\Delta T_{superheat} = 5\ ^oC$, $p_2 = p_3 = 800$ kPa, $\Delta T_{supcooling} = 5\ ^oC$, $Q_L = 300$ kJ/min

- <u>Continue Solution:</u>
  - » Coefficient of Performance cooling:

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\Rightarrow COP_R = \frac{\eta_C (h_1 - h_4)}{h_{2s} - h_1}$$

$$COP_R = 3.425$$



  - » Power input:

$$\dot{W}_{in} = \dot{m}(h_2 - h_1)$$

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

Example 2

$p_1 = p_4 = 140$ kPa, $\Delta T_{superheat} = 5\ ^oC$, $p_2 = p_3 = 800$ kPa, $\Delta T_{supcooling} = 5\ ^oC$, $Q_L = 300$ kJ/min

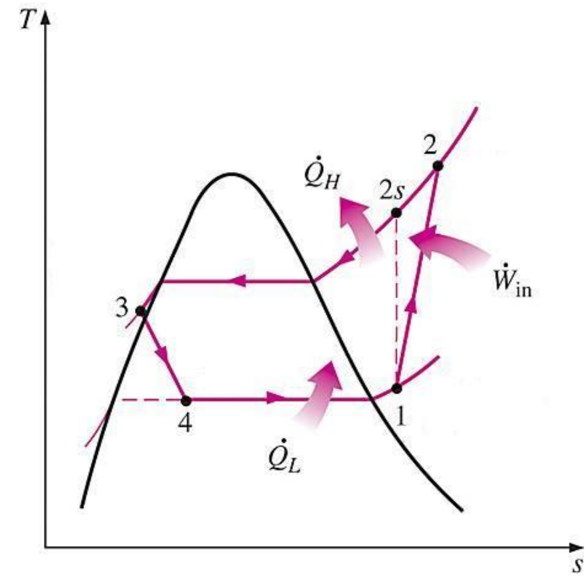
- <u>Continue Solution:</u>
  - » Power input:

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)}$$

$$\dot{W}_{in} = \frac{\dot{Q}_{in}}{(h_1 - h_4)}(h_2 - h_1)$$

$$\dot{W}_{in} = \frac{\dot{Q}_{in}}{COP_R} = \frac{300\ {}^{kJ}\!/_{min}}{3.425}\left(\frac{1\min}{60s}\right)$$

$$\dot{W}_{in} = 1.46kW$$



Cycle Analysis

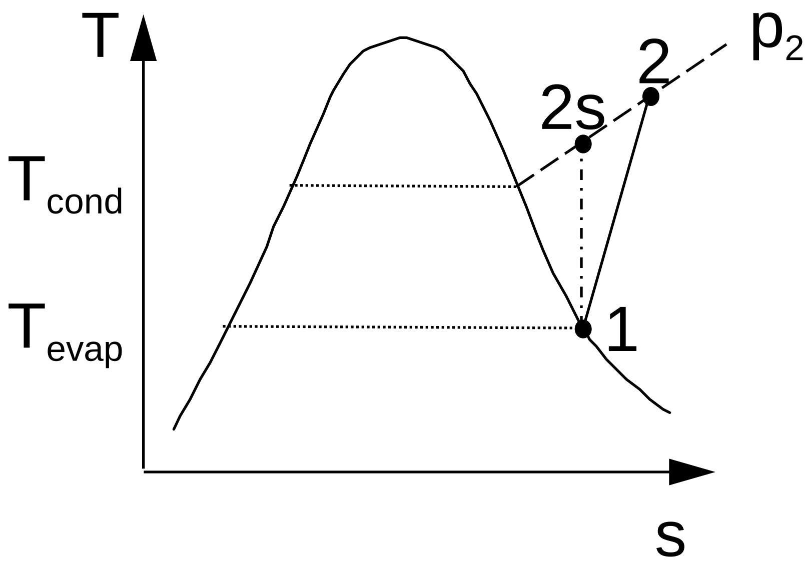
Compressor Inlet State

$p_1 = p_{evap}$ $\qquad$ $h_1 = h\,(T_{evap} + \Delta T_{sh}\ ,\ p_{evap})$

Compressor Outlet State

$p_2 = p_{cond}$

$$h_2 = h_1 + \frac{\overbrace{(h_{2s} - h_1)}^{\text{isentropic specific work}}}{\underbrace{\eta_s}_{\text{isentropic efficiency}}}$$



$h_{2s} = h\,(p_2\ ,\ s_2 = s_1)$

Reading material

Cycle Analysis

Condenser Outlet State

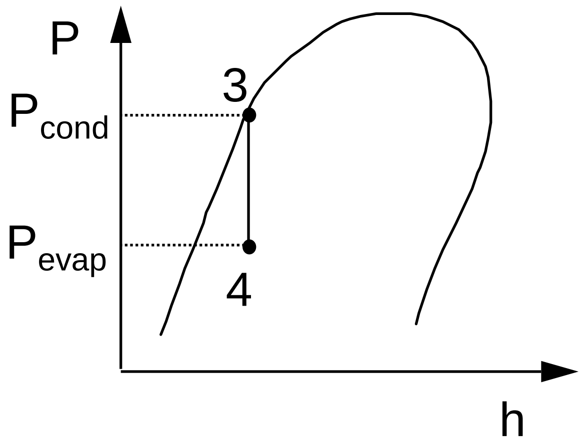
$$p_3 = p_{cond}$$

$$h_3 = h\,(T_{cond}+\Delta T_{sc}\,,\,p_{cond})$$

Evaporator Inlet State

$$p_4 = p_{evap}$$

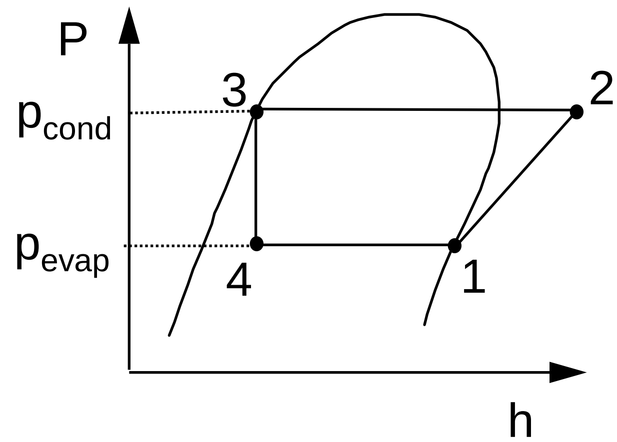
$$h_4 = h_3$$



Cycle Analysis

Coefficient of Performance

$$COP = \frac{\overbrace{h_1 - h_4}^{\text{refrigerating effect}}}{\underbrace{h_2 - h_1}_{\text{specific work}}}$$



Volumetric Capacity

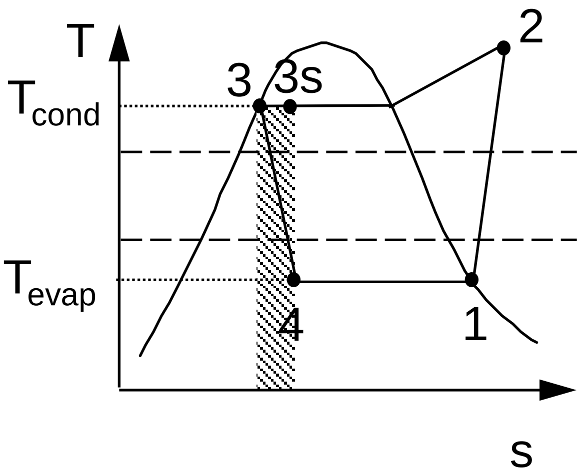
$$q_v = \frac{\overbrace{h_1 - h_4}^{\text{refrigerating effect}}}{\underbrace{v_1}_{\substack{\text{compressor inlet} \\ \text{specific volume}}}}$$

indicator of the compressor displacement requirement

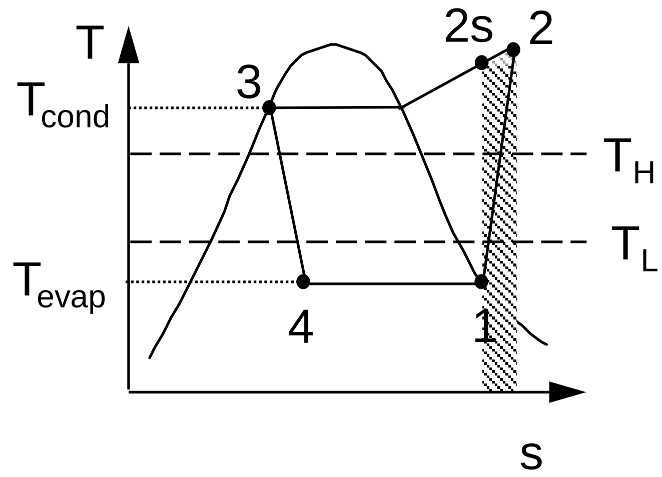
Cycle Analysis

Sources for Irreversibilities

Throttling Losses



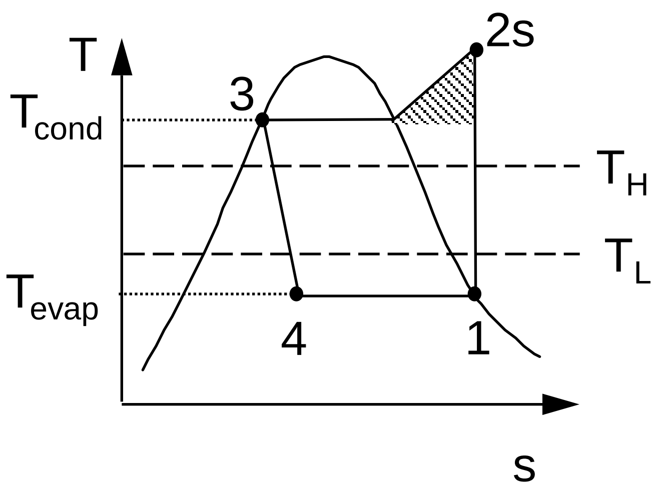
Compressor Losses



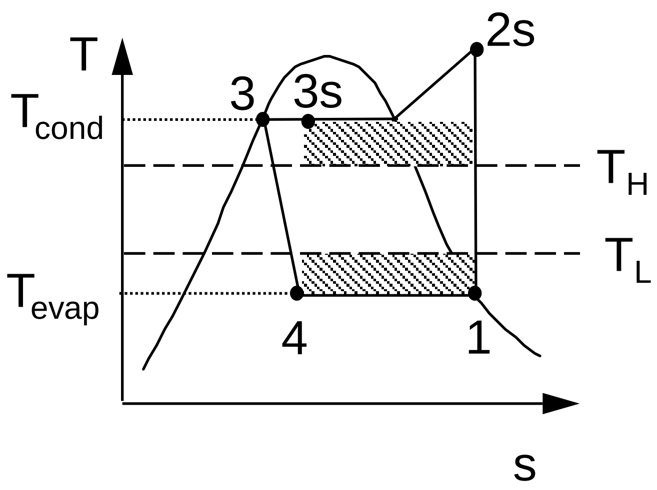
Cycle Analysis

Sources for Irreversibilities

Desuperheating Losses



Heat Exchange Losses



» also includes losses due to pressure drops in heat exchangers and distribution piping