Lecture 17-18: Illustrations_1: Nozzles, Diffusers, Turbines, Compressors

Yong Li
Shanghai Jiao Tong University
Institute of Refrigeration and Cryogenics
800 Dong Chuan Road Shanghai, 200240, P. R. China
Email: liyo@sjtu.edu.cn
Phone: 86-21-34206056; Fax: 86-21-34206056
Lecture Contents

● Last lecture---Open system (Control Volume system)
  Mass balance

\[
\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

\[\dot{m}_1 = \dot{m}_2\]

Energy balance

\[
dE_{cv} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

Steady state

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]
\]

● This lecture---Illustrations
  » Modeling Control Volumes at Steady State
  » Nozzles, diffusers, turbines
  » Compressors, pumps
Modeling Control Volumes at Steady State

- The CV is **modeled by making assumptions**.
- The step of **listing assumptions** is necessary in every **engineering analysis**.
- **Steady-state** assumption apply when
  - properties **fluctuate slightly** about their averages
  - periodic time variations
- There is no net change in the total energy and the total mass within the control volume.
- The time-averaged mass flow rates, heat transfer rates, work rates, and properties of the substances crossing the control surface all remain constant.
Modeling Control Volumes at Steady State

\[ \frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_{i} \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{e} \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) \]

- \[ \dot{Q}_{cv} = 0 \] in the energy rate balance because it is small relative to other energy transfers across the boundary.  
Valid when:

- The outer surface of the CV is well insulated.
- The outer surface area is too small for there to be effective heat transfer.
- The \( \Delta T \) between the CV and its surroundings is so small that the heat transfer can be ignored.
- The gas or liquid passes through the CV so quickly that there is not enough time for significant heat transfer to occur.
Modeling Control Volumes at Steady State

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]

- \( W_{cv} = 0 \) when there are no rotating shafts, displacements of the boundary, electrical effects, or other work mechanisms associated with the CV being considered.
- The KE and PE of the matter entering and exiting the control volume are neglected when they are small relative to other energy transfers.

Steady State

Steady flow SSSF

\[ \dot{m} = \text{const} \]

Steady State

\[ \dot{Q}_{cv} = \text{const} \quad \dot{W} = \text{const} \quad \dot{m} = \text{const} \]
CONCEPTS

Nozzle & Diffuser

- **Nozzle ::::** a gas or liquid flow passage of varying cross-sectional area in which the $V \uparrow$ in the direction of flow.

- **Diffuser ::::** a gas or liquid flow passage of varying cross-sectional area in which the $V \downarrow$ in the direction of flow.
Features of Nozzle & Diffuser

● In a nozzle, \( V \uparrow \) while \( p \downarrow \) decreases from inlet to exit (\( v \uparrow \))

● In a diffuser, \( V \downarrow \) while \( p \uparrow \) from inlet to exit (\( v \downarrow \))

● For nozzles and diffusers
  » The only work is flow work at locations where mass enters and exits the control volume
  » Heat transfer always negligible
    
    Small area; less time available for heat transfer due to
    Large velocities; small temperature difference
  » PE changes usually negligible
Applications

Wind-tunnel Test Facility

Co-annular

Convergent

Convergent–Divergent (CD)

Maneuvering

External Geometry

Nozzle

Inlet

Compressor

Burner

Turbine

Flow-straightening screens

Acceleration

Test section

Deceleration

Diffuser

1.8
Analysis

- At steady state the mass and energy rate balances

\[
\frac{d\dot{m}_{cv}}{dt} = \dot{m}_1 - \dot{m}_2
\]

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left( h_1 + \frac{V_1^2}{2} + g z_1 \right) - \dot{m}_2 \left( h_2 + \frac{V_2^2}{2} + g z_2 \right)
\]

- where 1 denotes the inlet and 2 the exit.

\[
0 = \frac{\dot{Q}_{cv}}{n} + (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right)
\]
Example 1: Deceleration of Air in a Diffuser

- Air at 10° C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m². The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine

  (a) the mass flow rate of the air.

  (b) the temperature of the air leaving the diffuser.

\[
\begin{align*}
\nu_1 &= \frac{RT_1}{P_1} = \frac{0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}}{80 \text{ kPa}} \\
&= 1.015 \text{ m}^3/\text{kg}
\end{align*}
\]

\[
\dot{m} = \frac{1}{\nu_1} V_1 A_1 = \frac{1}{1.015 \text{ m}^3/\text{kg}} (200 \text{ m/s})(0.4 \text{ m}^2) = 78.8 \text{ kg/s}
\]
Example 1 continued

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]

\[ \dot{Q} \approx 0, \dot{W} = 0, \text{ and } \Delta p e \approx 0 \]

\[ \dot{m} \left( h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left( h_2 + \frac{V_2^2}{2} \right) \]

\[ h_2 = h_1 - \frac{V_2^2 - V_1^2}{2} \]

\[ V_2 \ll V_1 \]

\[ h_1 = h @ 283 \text{ K} = 283.14 \text{ kJ/kg} \]

\[ h_2 = 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 303.14 \text{ kJ/kg} \]

\[ \text{From Table} \quad T_2 = 303 \text{ K} \]

\[ P_1 = 80 \text{ kPa} \]

\[ T_1 = 10^\circ \text{C} \]

\[ V_1 = 200 \text{ m/s} \]

\[ A_1 = 0.4 \text{ m}^2 \]
Example 2 Calculating Exit Area of a Steam Nozzle

- Find: The exit area.
- Assumptions:
  - steady state
  - \( \dot{Q}_\text{ev} = 0 \), \( \dot{W}_\text{ev} = 0 \)
  - \( \Delta \text{PE} = 0 \)
Example 2 continued

\[0 = \dot{Q}_{cv}^0 - \dot{W}_{cv}^0 + \dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) - \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2)\]

\[0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\]

\[h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)\]

- From Table A-4, \(h_1 = 3213.6 \text{ kJ/kg}\)

\[h_2 = 3213.6 \text{ kJ/kg} + \left[\frac{(10)^2 - (665)^2}{2}\right] \left(\frac{\text{m}^2}{\text{s}^2}\right) \left|\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}}\right|\]

\[= 3213.6 - 221.1 = 2992.5 \text{ kJ/kg}\]

- Table A-4 at \(p_2 = 15 \text{ bar}\), \(h_2 = 2992.5 \text{ kJ/kg}\)

\[\nu_2 = 0.1627 \text{ m}^3/\text{kg}\]

\[A_2 = \frac{\dot{m}v_2}{V_2} = \frac{(2 \text{ kg/s})(0.1627 \text{ m}^3/\text{kg})}{665 \text{ m/s}} = 4.89 \times 10^{-4} \text{ m}^2\]
Turbines

- **Turbine**: a steady-flow engineering device used to produce work output by expansion process

- **Features**
  - In a turbine, \( v \uparrow \) from inlet to exit as the working fluid expands
  - Work interaction device – desired output mechanical shaft work due to expansion
  - \( \dot{Q}_{cv} = 0 \)
  - \( \Delta PE = 0 \)
  - May need to consider KE changes if given
Applications

Hydraulic turbine installed in a dam.

- Turbines are widely used in vapor power plants, gas turbine power plants, and aircraft engines (Chaps. 8 and 9).
Gas Turbine

- A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.
Analysis

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \]

\[ \dot{m} \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{out} + \dot{m} \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) \]
Example 3 **Power Generation by a Steam Turbine**

- The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in figure.

(a) Compare the magnitudes of $\Delta h$, $\Delta ke$, and $\Delta pe$.

(b) Determine the work done per unit mass of the steam flowing through the turbine.

(c) Calculate the mass flow rate of the steam.

**Known:** The inlet and exit conditions of a steam turbine and its power output are given.

**Find:** The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.
Example 3

(a) Compare the magnitudes of $\Delta h$, $\Delta ke$, and $\Delta pe$

(b) work done per unit mass of the steam

(c) mass flow rate of the steam.

Assumptions: steady-flow process, The system is adiabatic

\[ \dot{m}_{cv} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \]

\[ \frac{dE_{cv}}{dt} = C_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) \]

<table>
<thead>
<tr>
<th>$P_1 = 2$ MPa</th>
<th>$T_1 = 400^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1 = 3248.4$ kJ/kg</td>
<td></td>
</tr>
</tbody>
</table>

\[ h_2 = h_f + x_2h_{fg} = \left[ 225.94 + (0.9)(2372.3) \right] \text{kJ/kg} = 2361.01 \text{ kJ/kg} \]

\[ \Delta h = h_2 - h_1 = (2361.01 - 3248.4) \text{ kJ/kg} = -887.39 \text{ kJ/kg} \]

\[ \Delta ke = \frac{V_2^2 - V_1^2}{2} = \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 14.95 \text{ kJ/kg} \]

\[ \dot{m} \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{out} + \dot{m} \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) \]

\[ \dot{W}_{out} = - \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = - (\Delta h + \Delta ke + \Delta pe) \]

\[ = -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = 872.48 \text{ kJ/kg} \]

\[ \dot{m} = \frac{\dot{W}_{out}}{\dot{W}_{out}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = 5.73 \text{ kg/s} \]

STEAM TURBINE

- $P_2 = 15$ kPa
- $x_2 = 90\%$
- $V_2 = 180 \text{ m/s}$
- $z_2 = 6 \text{ m}$

$\dot{W}_{out} = 5 \text{ MW}$
Compressor and Pumps

- **Compressor**: device in which work is done on a gas passing through them in order to *raise the pressure*.
- **Pumps**: the work input is used to *raise the pressure* of a liquid.

### Concepts

- **Reciprocating compressor**
- **Rotating compressors**
  - (a) Axial flow.
  - (b) Centrifugal.
  - (c) Roots type
Compressor and Pumps

- Work interaction device – desired output increase in pressure due to compression

- In a compressor, $\nu \downarrow$ from inlet to exit as the working fluid is compressed

- In a pump, $\Delta \nu \cong 0$

- $\dot{Q}_{cv} = 0$

- $\Delta PE \cong 0$

- may need to consider KE changes if given
Example 4 Compressing Air by a Compressor

- Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Determine the necessary power input to the compressor.

- Assumptions: 1. steady-flow process, 2. Air is an ideal gas, 3. $\Delta KE = \Delta PE = 0$

\[ \dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \]

\[ \dot{W}_{in} = \dot{m}q_{out} + \dot{m}(h_2 - h_1) \]

\[ h_1 = h @ 280 \text{ K} = 280.13 \text{ kJ/kg} \]

\[ h_2 = h @ 400 \text{ K} = 400.98 \text{ kJ/kg} \]

\[ \dot{W}_{in} = (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg} \]

\[ = 2.74 \text{ kW} \]
Turbine and Compressor

1st Law, Open System
Steady-State, SISO

\[ Q - W_s = \dot{m} \left[ \Delta H + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c} \Delta z \right] \]

Only if
Adiabatic
Not
Always!

\[ W_s = -\dot{m} \Delta H \]

Turbines & Compressors

- \( Q \) and \( \Delta E_k \) are often, but not always negligible.
- Enthalpy increases, \( \Delta H > 0 \).
- \( P \) increases dramatically and \( T \) usually increases as well.

- \( W_s < 0 \)
- \( W_s > 0 \)
Summary

- Open system at **steady state one-inlet one-outlet** (Control Volume system)

### Mass balance

\[ \dot{m}_1 = \dot{m}_2 \]

### Energy balance

\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left( h_1 - h_2 \right) + \left( \frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2) \]

- **nozzles**
- **diffusers**

\[ \dot{Q}_{cv} = 0 \]
\[ \dot{W}_{cv} = 0 \]

\[ 0 = (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) \]

- **turbines**
- **compressors**
- **pumps**

\[ \dot{Q}_{cv} = 0 \]

\[ \dot{m} \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{out} + \dot{m} \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) \]